Basic Factory Dynamics

Physics should be explained as simply as possible, but no simpler.

– Albert Einstein

HAL Case

Large Panel Line: produces unpopulated printed circuit boards

Line runs 24 hr/day (but 19.5 hrs of productive time)

Recent Performance:
  • throughput = 1,400 panels per day (71.8 panels/hr)
  • WIP = 47,600 panels
  • CT = 34 days (663 hr at 19.5 hr/day)
  • customer service = 75% on-time delivery

Is HAL lean?

What data do we need to decide?
HAL - Large Panel Line Processes

Lamination (Cores): press copper and prepreg into core blanks
Machining: trim cores to size
Internal Circuitize: etch circuitry into copper of cores
Optical Test and Repair (Internal): scan panels optically for defects
Lamination (Composites): press cores into multiple layer boards
External Circuitize: etch circuitry into copper on outside of composites
Optical Test and Repair (External): scan composites optically for defects
Drilling: holes to provide connections between layers
Copper Plate: deposits copper in holes to establish connections
Procoat: apply plastic coating to protect boards
Sizing: cut panels into boards
End of Line Test: final electrical test

HAL Case - Science?

External Benchmarking
• but other plants may not be comparable

Internal Benchmarking
• capacity data: what is utilization?
• but this ignores WIP effects

Need relationships between WIP, TH, CT, service!
Definitions

Workstations: a collection of one or more identical machines.

Parts: a component, sub-assembly, or an assembly that moves through the workstations.

End Items: parts sold directly to customers; relationship to constituent parts defined in bill of material.

Consumables: bits, chemicals, gasses, etc., used in process but do not become part of the product that is sold.

Routing: sequence of workstations needed to make a part.

Order: request from customer.

Job: transfer quantity on the line.

Definitions (cont.)

Throughput (TH): for a line, throughput is the average quantity of good (non-defective) parts produced per unit time.

Work in Process (WIP): inventory between the start and endpoints of a product routing.

Raw Material Inventory (RMI): material stocked at beginning of routing.

Crib and Finished Goods Inventory (FGI): crib inventory is material held in a stockpoint at the end of a routing; FGI is material held in inventory prior to shipping to the customer.

Cycle Time (CT): time between release of the job at the beginning of the routing until it reaches an inventory point at the end of the routing.
**Factory Physics®**

**Definition:** A manufacturing system is a goal-oriented network of processes through which parts flow.

**Structure:** Plant is made up of routings (lines), which in turn are made up of processes.

**Focus:** Factory Physics® is concerned with the network and flows at the routing (line) level.

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**Parameters**

**Descriptors of a Line:**

1) **Bottleneck Rate** ($r_b$): Rate (parts/unit time or jobs/unit time) of the process center having the highest long-term utilization.

2) **Raw Process Time** ($T_0$): Sum of the long-term average process times of each station in the line.

3) **Congestion Coefficient** ($\alpha$): A unitless measure of congestion.
   - Zero variability case, $\alpha = 0$.
   - "Practical worst case," $\alpha = 1$.
   - "Worst possible case," $\alpha = W_0$.

*Note:* we won’t use $\alpha$ quantitatively, but point it out to recognize that lines with same $r_b$ and $T_0$ can behave very differently.
Parameters (cont.)

Relationship:

**Critical WIP (W₀):** WIP level in which a line having no congestion would achieve maximum throughput (i.e., rₒ) with minimum cycle time (i.e., T₀).

\[ W₀ = rₒ T₀ \]

The Penny Fab

**Characteristics:**
- Four identical tools in series.
- Each takes 2 hours per piece (penny).
- No variability.
- CONWIP job releases.

**Parameters:**

\[
\begin{align*}
rₒ & = 0.5 \text{ pennies/hour} \\
T₀ & = 8 \text{ hours} \\
W₀ & = 0.5 \times 8 = 4 \text{ pennies} \\
\alpha & = 0 \text{ (no variability, best case conditions)}
\end{align*}
\]
The Penny Fab

Time = 0 hours

The Penny Fab (WIP=1)

Time = 0 hours
The Penny Fab (WIP=1)

Time = 2 hours

The Penny Fab (WIP=1)

Time = 4 hours
The Penny Fab (WIP=1)

Time = 6 hours

The Penny Fab (WIP=1)

Time = 8 hours
The Penny Fab (WIP=1)

Time = 10 hours

The Penny Fab (WIP=1)

Time = 12 hours
The Penny Fab (WIP=1)

Time = 14 hours

The Penny Fab (WIP=1)

Time = 16 hours
## Penny Fab Performance

<table>
<thead>
<tr>
<th>WIP</th>
<th>TH</th>
<th>CT</th>
<th>TH×CT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.125</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### The Penny Fab (WIP=2)

Time = 0 hours
The Penny Fab (WIP=2)

Time = 2 hours

The Penny Fab (WIP=2)

Time = 4 hours
The Penny Fab (WIP=2)

Time = 6 hours

The Penny Fab (WIP=2)

Time = 8 hours
The Penny Fab (WIP=2)

Time = 10 hours

The Penny Fab (WIP=2)

Time = 12 hours
The Penny Fab (WIP=2)

Time = 14 hours

The Penny Fab (WIP=2)

Time = 16 hours
The Penny Fab (WIP=2)

Time = 18 hours

Penny Fab Performance

<table>
<thead>
<tr>
<th>WIP</th>
<th>TH</th>
<th>CT</th>
<th>TH×CT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.125</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.250</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The Penny Fab (WIP=4)

Time = 0 hours

The Penny Fab (WIP=4)

Time = 2 hours
The Penny Fab (WIP=4)

Time = 4  hours

The Penny Fab (WIP=4)

Time = 6  hours
The Penny Fab (WIP=4)

Time = 8 hours

The Penny Fab (WIP=4)

Time = 10 hours
The Penny Fab (WIP=4)

Time = 12 hours

The Penny Fab (WIP=4)

Time = 14 hours
Penny Fab Performance

<table>
<thead>
<tr>
<th>WIP</th>
<th>TH</th>
<th>CT</th>
<th>TH×CT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.125</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.250</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.375</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0.500</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Penny Fab (WIP=5)

Time = 0 hours
The Penny Fab (WIP=5)

Time = 2 hours

The Penny Fab (WIP=5)

Time = 4 hours
The Penny Fab (WIP=5)

Time = 6 hours

The Penny Fab (WIP=5)

Time = 8 hours
The Penny Fab (WIP=5)

Time = 10 hours

The Penny Fab (WIP=5)

Time = 12 hours
## Penny Fab Performance

<table>
<thead>
<tr>
<th>WIP</th>
<th>TH</th>
<th>CT</th>
<th>TH×CT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.125</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.250</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.375</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0.500</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>0.500</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>0.500</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

## TH vs. WIP: Best Case

![TH vs. WIP: Best Case](image_url)
CT vs. WIP: Best Case

Best Case Performance

**Best Case Law:** The minimum cycle time \( (CT_{\text{best}}) \) for a given WIP level, \( w \), is given by

\[
CT_{\text{best}} = \begin{cases} 
T_0, & \text{if } w \leq W_0 \\
\frac{w}{r_b}, & \text{otherwise.}
\end{cases}
\]

The maximum throughput \( (TH_{\text{best}}) \) for a given WIP level, \( w \) is given by,

\[
TH_{\text{best}} = \begin{cases} 
\frac{w}{T_0}, & \text{if } w \leq W_0 \\
\frac{1}{r_b}, & \text{otherwise.}
\end{cases}
\]
Best Case Performance (cont.)

Example: For Penny Fab, $r_b = 0.5$ and $T_0 = 8$, so $W_0 = 0.5 \times 8 = 4$,

$$CT_{best} = \begin{cases} 8, & \text{if } w \leq 4 \\ 2w, & \text{otherwise.} \end{cases}$$

$$TH_{best} = \begin{cases} w/8, & \text{if } w \leq 4 \\ 0.5, & \text{otherwise.} \end{cases}$$

which are exactly the curves we plotted.

A Manufacturing Law

**Little's Law:** The fundamental relation between WIP, CT, and TH over the long-term is:

$$WIP = TH \times CT$$

$$\text{parts} = \frac{\text{parts}}{hr} \times hr$$

Insights:

- Fundamental relationship
- Simple units transformation
- Definition of cycle time (CT = WIP/TH)
### Penny Fab Two

<table>
<thead>
<tr>
<th>Station Number</th>
<th>Number of Machines</th>
<th>Process Time</th>
<th>Station Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2 hr</td>
<td>0.5 j/hr</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>5 hr</td>
<td>0.4 j/hr</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>10 hr</td>
<td>0.6 j/hr</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3 hr</td>
<td>0.67 j/hr</td>
</tr>
</tbody>
</table>

\[
r_s = 0.4 \text{ p/hr} \quad T_0 = 20 \text{ hr} \quad W_0 = 8 \text{ pennies}
\]
Penny Fab Two Simulation (Time=4)

10 hr
2 hr
5 hr
3 hr
10 hr

Penny Fab Two Simulation (Time=6)

10 hr
2 hr
5 hr
3 hr
10 hr
Penny Fab Two Simulation (Time=7)

8
2 hr

12
5 hr

9
3 hr


17
10 hr

Penny Fab Two Simulation (Time=8)

10
2 hr

12
5 hr

9
3 hr


17
10 hr
Penny Fab Two Simulation (Time=12)

10 hr
2 hr
5 hr
3 hr
10 hr

Penny Fab Two Simulation (Time=14)

16 hr
2 hr
5 hr
3 hr
10 hr
Penny Fab Two Simulation (Time=19)

Note: job will arrive at bottleneck just in time to prevent starvation.

Penny Fab Two Simulation (Time=20)
Penny Fab Two Simulation (Time=22)

27
27
24

2 hr
5 hr
10 hr

24
24
25

Note: job will arrive at bottleneck just in time to prevent starvation.

Penny Fab Two Simulation (Time=24)

27
27
29

2 hr
5 hr
10 hr

24
24
25

And so on.... Bottleneck will just stay busy; all others will starve periodically
Worst Case

**Observation:** The Best Case yields the minimum cycle time and maximum throughput for each WIP level.

**Question:** What conditions would cause the *maximum* cycle time and *minimum* throughput?

**Experiment:**
- set average process times same as Best Case (so $r_b$ and $T_0$ unchanged)
- follow a marked job through system
- imagine marked job experiences maximum *queueing*

---

Worst Case Penny Fab

Time = 0 hours
Worst Case Penny Fab

Time = 8 hours

Worst Case Penny Fab

Time = 16 hours
Worst Case Penny Fab

Time = 24 hours

Worst Case Penny Fab

Time = 32 hours

Note:

\[ CT = 32 \text{ hours} \]
\[ = 4 \times 8 = wT_0 \]
\[ TH = 4/32 = 1/8 = 1/T_0 \]
TH vs. WIP: Worst Case

CT vs. WIP: Worst Case
Worst Case Performance

Worst Case Law: The worst case cycle time for a given WIP level, \( w \), is given by,

\[
CT_{\text{worst}} = wT_0
\]

The worst case throughput for a given WIP level, \( w \), is given by,

\[
TH_{\text{worst}} = \frac{1}{T_0}
\]

Randomness? None - perfectly predictable, but bad!

Practical Worst Case

Observation: There is a **BIG GAP** between the Best Case and Worst Case performance.

Question: Can we find an intermediate case that:
- divides “good” and “bad” lines, and
- is computable?

Experiment: consider a line with a given \( r_b \) and \( T_0 \) and:
- single machine stations
- balanced lines
- variability such that all WIP configurations (states) are equally likely
PWC Example – 3 jobs, 4 stations

<table>
<thead>
<tr>
<th>State</th>
<th>Vector</th>
<th>State</th>
<th>Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(3,0,0,0)</td>
<td>11</td>
<td>(1,0,2,0)</td>
</tr>
<tr>
<td>2</td>
<td>(0,3,0,0)</td>
<td>12</td>
<td>(0,1,2,0)</td>
</tr>
<tr>
<td>3</td>
<td>(0,0,3,0)</td>
<td>13</td>
<td>(0,0,2,1)</td>
</tr>
<tr>
<td>4</td>
<td>(0,0,0,3)</td>
<td>14</td>
<td>(1,0,0,2)</td>
</tr>
<tr>
<td>5</td>
<td>(2,1,0,0)</td>
<td>15</td>
<td>(0,1,0,2)</td>
</tr>
<tr>
<td>6</td>
<td>(2,0,1,0)</td>
<td>16</td>
<td>(0,0,1,2)</td>
</tr>
<tr>
<td>7</td>
<td>(2,0,0,1)</td>
<td>17</td>
<td>(1,1,1,0)</td>
</tr>
<tr>
<td>8</td>
<td>(1,2,0,0)</td>
<td>18</td>
<td>(1,1,0,1)</td>
</tr>
<tr>
<td>9</td>
<td>(0,2,1,0)</td>
<td>19</td>
<td>(1,0,1,1)</td>
</tr>
<tr>
<td>10</td>
<td>(0,2,0,1)</td>
<td>20</td>
<td>(0,1,1,1)</td>
</tr>
</tbody>
</table>

Note: average WIP at any station is 15/20 = 0.75, so jobs are spread evenly between stations.

Practical Worst Case

Let \( w \) = jobs in system, \( N \) = no. stations in line, and \( t \) = process time at all stations:

\[
CT(\text{single}) = (1 + (w-1)/N) t
\]

\[
CT(\text{line}) = N [1 + (w-1)/N] t = Nt + (w-1)t = T_0 + (w-1)/r_b
\]

\[
TH = \frac{WIP}{CT} \quad \text{From Little’s Law}
\]

\[
= \frac{w/(w+\sqrt{w^2-1})}{r_b}
\]
Practical Worst Case Performance

**Practical Worst Case Definition:** The practical worst case (PWC) cycle time for a given WIP level, \( w \), is given by,

\[
CT_{\text{PWC}} = T_0 + \frac{w - 1}{r_b}
\]

The PWC throughput for a given WIP level, \( w \), is given by,

\[
TH_{\text{PWC}} = \frac{w}{W_0 + w - 1} r_b
\]

where \( W_0 \) is the critical WIP.

---

**TH vs. WIP: Practical Worst Case**

![TH vs. WIP graph](http://www.factory-physics.com)
CT vs. WIP: Practical Worst Case

- **Worst Case**
- **PWC**
- **Bad (fat)**
- **Good (lean)**

Best Case

Worst Case

PWCPWC

Note: process times in PF2 have var equal to PWC.

But… unlike PWC, it has unbalanced line and multi machine stations.
Back to the HAL Case - Capacity Data

<table>
<thead>
<tr>
<th>Process</th>
<th>Rate (p/hr)</th>
<th>Time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamination</td>
<td>191.5</td>
<td>1.2</td>
</tr>
<tr>
<td>Machining</td>
<td>186.2</td>
<td>5.9</td>
</tr>
<tr>
<td>Internal Circuitize</td>
<td>150.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Optical Test/Repair - Int</td>
<td>157.8</td>
<td>5.6</td>
</tr>
<tr>
<td>Lamination – Composites</td>
<td>191.5</td>
<td>1.2</td>
</tr>
<tr>
<td>External Circuitize</td>
<td>150.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Optical Test/Repair - Ext</td>
<td>157.8</td>
<td>5.6</td>
</tr>
<tr>
<td>Drilling</td>
<td>185.9</td>
<td>10.0</td>
</tr>
<tr>
<td>Copper Plate</td>
<td>136.4</td>
<td>1.5</td>
</tr>
<tr>
<td>Procoat</td>
<td>146.2</td>
<td>2.2</td>
</tr>
<tr>
<td>Sizing</td>
<td>126.5</td>
<td>2.4</td>
</tr>
<tr>
<td>EOL Test</td>
<td>169.5</td>
<td>1.8</td>
</tr>
<tr>
<td>$r_s, T_0$</td>
<td>126.5</td>
<td>33.1</td>
</tr>
</tbody>
</table>
HAL Case - Situation

Critical WIP: $W_0 = r_b T_0 = 126.5 \times 33.9 = 4,187$

Actual Values:
- $CT = 34$ days = 816 hours (at 24 hr/day)
- $WIP = 37,000$ panels
- $TH = 45.8$ panels/hour

Conclusions:
- Throughput is 36% of capacity
- WIP is 15 times critical WIP
- $CT$ is 24.6 times raw process time

HAL Case - Analysis

TH Resulting from PWC with $WIP = 47,600$?

$$TH = \frac{w}{w + W_0 - 1} r_b = \frac{37,400}{37,400 + 4,187 - 1} 126.5 = 105.4$$

WIP Required for PWC to Achieve $TH = 0.63 r_b$?

$$TH = \frac{w}{w + W_0 - 1} r_b = 0.36 r_b$$

$$w = \frac{0.36 (W_0 - 1)}{0.64} = \frac{0.36 (4,187 - 1)}{0.64} = 2,354$$

Conclusion: actual system is much worse than PWC!
**Labor Constrained Systems**

**Motivation:** performance of some systems are limited by labor or a combination of labor and equipment.

**Full Flexibility with Workers Tied to Jobs:**
- WIP limited by number of workers ($n$)
- capacity of line is $n/T_0$
- Best case achieves capacity and has workers in “zones”
- *ample capacity* case also achieves full capacity with “pick and run” policy
Labor Constrained Systems (cont.)

Full Flexibility with Workers Not Tied to Jobs:
• TH depends on WIP levels
• $TH_{CW}(w) \leq TH(w) \leq TH_{CW}(w)$
• need policy to direct workers to jobs (focus on downstream is effective)

Agile Workforce Systems
• bucket brigades
• kanban with shared tasks
• worksharing with overlapping zones
• many others

Factory Dynamics Takeaways

Performance Measures:
• throughput
• WIP
• cycle time
• service

Range of Cases:
• best case
• practical worst case
• worst case

Diagnostics:
• simple assessment based on $r_b, T_p$, actual WIP, actual TH
• evaluate relative to practical worst case