The Matrix Exponential on Time Scales

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Abstract

Mathematical models of many physical, biological, and economic processes involve systems of linear, constant coefficient differential equations

$$x' = Ax$$

or difference equations

$$x(n+1) = Ax(n),$$

where A is a given, fixed, real or complex $n \times n$ matrix. A solution vector x is sought which satisfies an initial condition $x(0) = x_0$. In control theory, A is known as the state companion matrix and x is the system response. In principle, the matrix exponential e^{tA} and the matrix A^k for any given $n \times n$ matrix A can be computed in many ways. The methods involve approximation theory, differential/difference equations, the matrix eigenvalues, and the matrix characteristic polynomial. The matrix exponential can be found in various connections in analysis and control of dynamic systems. Following Hilger's landmark paper [S.Hilger, Analysis on measure chains – a unified approach to continuous and discrete calculus, *Results Math.*, 18 (1990) 18–56.], a rapidly expanding body of literature has sought to unify, extend, and generalize ideas from discrete calculus and continuous calculus to arbitrary time-scale calculus. A time scale $\mathbb{T}$ is simply any nonempty closed set of real numbers. In this talk I will introduce some basics of time scale calculus and present an elementary method for calculating the matrix exponential on an arbitrary time scale, thus including as corollaries the matrix exponential in differential equations, difference equations, q-difference equations, and others.