Boundedness And Exponential Stability In Highly Nonlinear Stochastic Differential Equations

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Let $B(t) = (B_1(t), B_2(t), \dots, B_m(t))^T$ be a m-dimensional standard Brownian motion defined on a complete probability space $(\Omega, \mathfrak{F}, \mathrm{P})$. Consider n-dimensional stochastic systems

$$dx(t) = f(x(t), t)dt + g(x(t), t)dB(t),\ t \geq 0, \tag{1}$$

with initial condition $x(t_0) = x_0 \in \mathbb{R}^n$, where $t_0 \geq 0$,
$x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{R}^n$, and $f : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^{n\times m}$ are given nonlinear continuous functions.
It is known that if the functions f and g satisfy a general Lipschitz condition and linear growth condition, then all solutions of system (1) exist stochastically.
In this research, we use the method of Lyapunov functions to obtain sufficient conditions for stochastic boundedness and exponential asymptotic stability of system (1) without the above requirement on the functions f and g.
Our theorems will make significant contribution to the theory of stochastic differential equations differential equations when dealing with equations that might contain unbounded terms. The theory is illustrated with several examples.