Izmir University of Economics
Department of Computer Engineering

CE401
Algorithm Design

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Final Exam
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Question 1 20 points. Given a 2-dimensional array $A[1..n,1..m]$ of integers, describe and analyze a dynamic programming algorithm that finds the largest product of elements in a contiguous 2d-subarray $A[i..j,k..l]$.

Question 2 20 points. You are given a set of $n$ types of cylinders, where the $i^{th}$ cylinder has height $h_i$, radius $r_i$, and a weight $w_i$. You want to create a stack of cylinders which is as tall as possible, but you can only stack a cylinder on top of another cylinder if the area of the base of the lower cylinder is strictly larger than the area of the base of the cylinders above it. Describe and analyze a dynamic programming algorithm that finds the stack of cylinders with maximum weight.

Question 3 20 points. Suppose we have $n$ pairs of identical twins. Each pair is denoted by $(i, i)$. We separate each pair $(i, i)$ of twins into two sequences $L$ and $R$. One twin $i$ goes to position $i$ in sequence $L$ and the other one to a random position in sequence $R$. Thus, $L$ will always have the twins in the following order $(1, \ldots, n)$ and $R$ could be any permutation of $L$. For instance, if $n = 4$, then $L$ is $(1, 2, 3, 4)$ and a possible $R$ could be $(2, 1, 3, 4)$.

Suppose we arrange the persons in sequence $L$ in a row and opposite to it are the persons in sequence $R$. Describe and analyze a dynamic programming algorithm that finds the maximum number of pairs of twins that can shake their hands without having any crossings. For instance, in the figure below is a solution of value 3.

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L:  1  2  3  4  5  6
R:  3  2  6  4  1  5
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Question 4 20 points. Consider two stacks $S$ and $T$ each having $n$ numbers. We play a game against an opponent by alternating turns. In each turn, a player selects either the top element $i$ of stack $S$ or the top element $j$ of stack $T$, removes it from the chosen stack permanently, and increments his score with it. The game ends when both stacks are empty. Determine the maximum possible score we can definitely win if we move first using a dynamic programming algorithm irrespective of what the other player does.

Note that each player knows the contents of both stacks throughout the game.

Question 5 20 points. Suppose you are given an $n \times m$ array, represented by an array $M[1..n,1..m]$ whose entries are one of the three colors $\{\text{red, blue, green}\}$. A solid block is a subarray of the form $M[i..j,k..l]$ in which all entries have the same color. Describe and analyze a dynamic programming algorithm to find a solid block in $M$ with maximum area.