1. (20 points) Design a PDA that recognizes the following language: \( \{0^i1^j | i > 0, j \geq 0 \} \)

2. (20 points)
   Design a PDA over \( \Sigma = \{a, b\} \) that recognizes the following language:
   \( L = \{a^n b^{2(n+m)} c^m | n, m \geq 0 \} \)

3. (20 points)
   Design a PDA over \( \Sigma = \{a, b\} \) that accepts all strings in which the number of a’s is equal to the number of b’s.

4. (20 points) Design a TM over \( \Sigma = \{0, 1\} \) that concatenates a string to itself. Assume that the input is given on the tape as \#w where w is the input string. You must output \#ww on the tape. Your TM must halt! Clearly indicate all details of your TM. You can assume that w is not empty.
   For example, given the initial tape of \#00110, the TM must halt with the following tape: \#0011000110

5. (20 points) Given the following transition function for Turing Machine \( T \):
   
   \[
   \begin{align*}
   (q_0, \#) &\rightarrow (q_1, \#, R) \\
   (q_0, a) &\rightarrow (q_0, a, R) \\
   (q_0, b) &\rightarrow (q_1, b, R) \\
   (q_1, a) &\rightarrow (q_1, a, R) \\
   (q_1, b) &\rightarrow (q_2, b, L) \\
   (q_2, a) &\rightarrow (q_2, b, L) \\
   (q_2, b) &\rightarrow (q_3, b, R) \\
   (q_3, a) &\rightarrow (q_0, a, R) \\
   (q_3, b) &\rightarrow (q_3, b, R) \\
   (q_3, \#) &\rightarrow (q_1, \#, R)
   \end{align*}
   \]
   The initial state is \( q_0 \) and the accept state is \( q_4 \).

   a) (10) Given the initial tape of \textbf{aababa}\# what is the final configuration of the tape?
   b) (10) Given the initial tape of \textbf{baabaabab}\# what is the final configuration of the tape?
   c) (10) What does this Turing Machine do in general?