1. (15 points)
   a) (10 points) Design a DFA over \( \Sigma = \{0, 1\} \) that accepts all strings that contain an even number of 0s and ends with 1.
   b) (5 points) Do it with at most 3 states.

2. (20 points)
   a) (15 points) Design a DFA over \( \Sigma = \{0, 1, 2\} \) that accepts any string which starts with 0, contains no 1s and ends with 2.
   b) (5 points) Write down the formal definition of your DFA.

   a) \( Q = \{q_0, q_1, q_2, q_3\} \)
   b) \( \Sigma = \{0, 1, 2\} \)
   c) \( \delta = \)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_0 )</td>
<td>( q_1 )</td>
<td>( q_3 )</td>
<td>( q_3 )</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>( q_1 )</td>
<td>( q_3 )</td>
<td>( q_2 )</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>( q_1 )</td>
<td>( q_3 )</td>
<td>( q_2 )</td>
</tr>
<tr>
<td>( q_3 )</td>
<td>( q_3 )</td>
<td>( q_3 )</td>
<td>( q_3 )</td>
</tr>
</tbody>
</table>
   d) \( q_0 \)
   e) \( F = \{q_2\} \)
3. (20 points)
   a) (10 points) Construct a DFA over $\Sigma = \{0, 1\}$ that accepts all strings that consist of a string of 0s followed by a string of 1s. For example 000011 or 01111 are both accepted. The following is a formal definition of the language:
   
   \[
   L_1 = \{0^i1^j \mid i, j \geq 0\}
   \]

   b) (10 points) Construct an NFA for the same language. Your NFA must not be a DFA!

4. (45 points)
   a) (15 points) Design an NFA over $\Sigma = \{0, 1\}$ that accepts language $L_2$ for which each string starts and ends with different symbols. That is, either the string starts with 1 and ends with 0, or it starts with 0 and ends with 1.

   For the following sub-questions use $L_1$ from the 3rd question:
   b) (10 points) Design an NFA that accepts $L_1 \cup L_2$. 
c) (10 points) Design an NFA that accepts $L_1 \circ L_2$.

d) (10 points) Design an NFA that accepts $L_2^*$. 