1. (15 points)
   a) (10 points) Design a DFA over $\Sigma = \{0, 1\}$ that accepts all strings that contain an even number of 0s and ends with 1.
   b) (5 points) Do it with at most 3 states.

2. (20 points)
   a) (15 points) Design a DFA over $\Sigma = \{0, 1, 2\}$ that accepts any string which starts with 0, contains no 1s and ends with 2.
   b) (5 points) Write down the formal definition of your DFA.

3. (20 points)
   a) (10 points) Construct a DFA over $\Sigma = \{0, 1\}$ that accepts all strings that consist of a string of 0s followed by a string of 1s. For example 000011 or 01111 are both accepted. The following is a formal definition of the language:
      
      \[ L_1 = \{0^i1^j \mid i, j \geq 0\} \]

   b) (10 points) Construct an NFA for the same language. Your NFA must not be a DFA!

4. (45 points)
   a) (15 points) Design an NFA over $\Sigma = \{0, 1\}$ that accepts language $L_2$ for which each string starts and ends with different symbols. That is, either the string starts with 1 and ends with 0, or it starts with 0 and ends with 1.
      
      For the following sub-questions use $L_1$ from the 3rd question:
   b) (10 points) Design an NFA that accepts $L_1 \cup L_2$.
   c) (10 points) Design an NFA that accepts $L_1 \circ L_2$.
   d) (10 points) Design an NFA that accepts $L_2$. 