Electromagnetic Signal

- Can be either analog or digital
- Function of time
- Can also be expressed as a function of frequency
  - Signal consists of components of different frequencies
Time-Domain Concepts

- Analog signal - signal intensity varies in a smooth fashion over time
  - No breaks or discontinuities in the signal
- Digital signal - signal intensity maintains a constant level for some period of time and then changes to another constant level
- Periodic signal - analog or digital signal pattern that repeats over time
  - $s(t + T) = s(t)$  $-\infty < t < +\infty$
  - where $T$ is the period of the signal
Time-Domain Concepts

- Aperiodic signal - analog or digital signal pattern that doesn't repeat over time
- Peak amplitude ($A$) - maximum value or strength of the signal over time; typically measured in volts
- Frequency ($f$)
  - Rate, in cycles per second, or Hertz (Hz) at which the signal repeats
Time-Domain Concepts

- **Period** ($T$) - amount of time it takes for one repetition of the signal
  - $T = 1/f$
- **Phase** ($\phi$) - measure of the relative position in time within a single period of a signal
- **Wavelength** ($\lambda$) - distance occupied by a single cycle of the signal
  - Or, the distance between two points of corresponding phase of two consecutive cycles
Sine Wave Parameters

- **General sine wave**
  \[ s(t) = A \sin(2\pi ft + \phi) \]

- Figure 2.3 shows the effect of varying each of the three parameters
  - (a) \( A = 1, \ f = 1 \ \text{Hz,} \ \phi = 0; \ \text{thus} \ T = 1\text{s} \)
  - (b) Reduced peak amplitude; \( A=0.5 \)
  - (c) Increased frequency; \( f = 2, \ \text{thus} \ T = \frac{1}{2} \)
  - (d) Phase shift; \( \phi = \frac{\pi}{4} \text{ radians (45 degrees)} \)

- note: \( 2\pi \text{ radians} = 360^\circ = 1 \text{ period} \)
Figure 2.3  \( s(t) = A \sin (2 \pi f t + \phi) \)
Time vs. Distance

- When the horizontal axis is *time*, as in Figure 2.3, graphs display the value of a signal at a given point in *space* as a function of *time*.

- With the horizontal axis in *space*, graphs display the value of a signal at a given point in *time* as a function of *distance*.

  - At a particular instant of time, the intensity of the signal varies as a function of distance from the source.
Frequency-Domain Concepts

- **Fundamental frequency** - when all frequency components of a signal are integer multiples of one frequency, it’s referred to as the fundamental frequency.

- **Spectrum** - range of frequencies that a signal contains.

- **Absolute bandwidth** - width of the spectrum of a signal.

- **Effective bandwidth** (or just bandwidth) - narrow band of frequencies that most of the signal’s energy is contained in.
Frequency-Domain Concepts

- Any electromagnetic signal can be shown to consist of a collection of periodic analog signals (sine waves) at different amplitudes, frequencies, and phases.
- The period of the total signal is equal to the period of the fundamental frequency.
Relationship between Data Rate and Bandwidth

- The greater the bandwidth, the higher the information-carrying capacity

Conclusions

- Any digital waveform will have infinite bandwidth
- BUT the transmission system will limit the bandwidth that can be transmitted
- AND, for any given medium, the greater the bandwidth transmitted, the greater the cost
- HOWEVER, limiting the bandwidth creates distortions
Spectrum and bandwidth

- Electromagnetic signals are made up of many frequencies
- Shown in the next example

\[ s(t) = \frac{4}{\pi} \left[ \sin(2\pi ft) + \frac{1}{3} \sin(2\pi(3f)t) \right] \]
FIG 1

Source: Stallings
Spectrum and bandwidth

- The 2\textsuperscript{nd} frequency is an integer multiple of the first frequency
  - When all of the frequency components of a signal are integer multiples of one frequency, the latter frequency is called \textit{fundamental frequency} (f)
  - \textit{Period} of the resultant signal is equal to the period of the fundamental frequency
    - Period of s(t) is T=1/f
Digital versus Analog Bandwidth

- The analog bandwidth $B$ is the difference between the highest and the lowest frequency required to send a composite signal.
  - Units: Hertz (Hz)
  - Recall that $1 \text{ Hz} = 1 \text{ s}^{-1}$
- The digital bandwidth ("bit rate") is based on the length of a bit in terms of the time taken to send the bit, measured in b/s.
- The two types of bandwidth are proportional to each other.
- The exact relationship depends on two factors:
  - The number of harmonics used in the analog signal (affects the analog bandwidth)
  - The number of signal levels used to represent a bit.
Digital versus Analog Bandwidth

Example:
- If the high and low amplitudes of a sine wave are used to encode a 1 and a 0, two bits can be encoded per sine wave period.
- Suppose we want to send 6000 b/s.
- One extreme: sending a long series of consecutive 0 (or 1) bits requires a signal with unvarying amplitude (i.e. frequency is 0 Hz).
- Other extreme: sending alternating 0 and 1 bits requires a signal of 3000 Hz. => 1 Hz represents 2 b/s.
- Analog bandwidth: B = 3000 Hz (3000 – 0)
- Digital bandwidth (“capacity”): C = 6000 b/s
- Relationship: B = C/2
- Adding harmonics increases B, so B ≥ n/2
Fourier Analysis

- Any signal is made up of components at various frequencies, in which each component is a sinusoid.
  - Adding enough sinusoidal signals with appropriate amplitude, frequency and phase, any electromagnetic signal can be constructed.
Spectrum and bandwidth

- It is the range of frequencies that a signal contains (among its components)
  - In the example, *spectrum is from f to 3f*
  - Absolute bandwidth is the width of the spectrum
    - \(3f-f = 2f\)
Data Rate and bandwidth

- There is a direct relationship between data rate (or signal carrying capacity) and bandwidth.
- Suppose we let a positive pulse represent 1 and a negative pulse represent 0.
  - Then the waveform (next slide) represents 1010..
  - Duration of each pulse is $t_{\text{bit}} = \frac{1}{2} \times \frac{1}{f}$
  - Thus data rate is $\frac{1}{t_{\text{bit}}} = 2f$ bits/sec
- As we add more and more frequencies the wave looks more like a square wave.
FIG 2
Example

- Looking at FIG 2(a) the bandwidth = 5f-f = 4f
  - If f=1MHz = 10^6 cycles/sec, then bandwidth = 4MHz
  - The period of the fundamental frequency = T = 1/f = 1 µs
  - So each bit takes up 0.5 µs i.e. data rate is 1/0.5 Mbps = 2 Mbps
Example

Looking at FIG 1(c) the bandwidth = 3f-f = 2f

- If f=2MHz = 2x10^6 cycles/sec, then bandwidth = 4MHz
- The period of the fundamental frequency = T = 1/f = 0.5 µs
- So each bit takes up 0.25 µs i.e. data rate is 1/0.25 Mbps = 4 Mbps
Example

- Thus a given bandwidth can support different data rate, depending on the ability of the receiver to discern the difference between 0 and 1 in the presence of noise and interference.
Data Communication Terms

- Data - entities that convey meaning, or information
- Signals - electric or electromagnetic representations of data
- Transmission - communication of data by the propagation and processing of signals
Examples of Analog and Digital Data

- Analog
  - Video
  - Audio
- Digital
  - Text
  - Integers
Analog Signals

- A continuously varying electromagnetic wave that may be propagated over a variety of media, depending on frequency

- Examples of media:
  - Copper wire media (twisted pair and coaxial cable)
  - Fiber optic cable
  - Atmosphere or space propagation

- Analog signals can propagate analog and digital data
Digital Signals

- A sequence of voltage pulses that may be transmitted over a copper wire medium
- Generally cheaper than analog signaling
- Less susceptible to noise interference
- Suffer more from attenuation
- Digital signals can propagate analog and digital data
Analog Signals: Represent data with continuously varying electromagnetic wave

Analog Data (voice sound waves) → Telephone → Analog Signal

Digital Data (binary voltage pulses) → Modem → Analog Signal (modulated on carrier frequency)
Digital Signals: Represent data with sequence of voltage pulses

Analog Signal -> Codec -> Digital Signal

Digital Data -> Digital Transmitter -> Digital Signal
Reasons for Choosing Data and Signal Combinations

- **Digital data, digital signal**
  - Equipment for encoding is less expensive than digital-to-analog equipment

- **Analog data, digital signal**
  - Conversion permits use of modern digital transmission and switching equipment

- **Digital data, analog signal**
  - Some transmission media will only propagate analog signals
  - Examples include optical fiber and satellite

- **Analog data, analog signal**
  - Analog data easily converted to analog signal
Analog Transmission

- Transmit analog signals without regard to content
- Attenuation limits length of transmission link
- Cascaded amplifiers boost signal’s energy for longer distances but cause distortion
  - Analog data can tolerate distortion
  - Introduces errors in digital data
Digital Transmission

- Concerned with the content of the signal
- Attenuation endangers integrity of data
- Digital Signal
  - Repeaters achieve greater distance
  - Repeaters recover the signal and retransmit
- Analog signal carrying digital data
  - Retransmission device recovers the digital data from analog signal
  - Generates new, clean analog signal
About Channel Capacity

- Impairments, such as noise, limit data rate that can be achieved.
- For digital data, to what extent do impairments limit data rate?
- Channel Capacity – the maximum rate at which data can be transmitted over a given communication path, or channel, under given conditions.
Concepts Related to Channel Capacity

- Data rate - rate at which data can be communicated (bps)
- Bandwidth - the bandwidth of the transmitted signal as constrained by the transmitter and the nature of the transmission medium (Hertz)
- Noise - average level of noise over the communications path
- Error rate - rate at which errors occur
  - Error = transmit 1 and receive 0; transmit 0 and receive 1
Channel Capacity

Four concepts:

- **Data Rate**: rate (in bps) at which data can be communicated.
- **Bandwidth**: bandwidth of the transmitted signal as constrained by the transmitter and the medium, expressed in Hz.
- **Noise**: interfering electromagnetic signal that tend to reduce the integrity of data signal.
- **Error rate**: rate at which receiver receives bits in error i.e. it receives a 0 when actually a 1 was sent and vice-versa.
Nyquist Bandwidth (assumes noise-free communication)

- Given a bandwidth of $B$, the highest signal rate that can be carried is $C = 2B$ (when signal transmitted is binary (two voltage levels))

- When $M$ voltage levels are used, then each signal level can represent $\log_2 M$ bits. Hence the Nyquist bandwidth (capacity) is given by
  
  $C = 2B \log_2 M$

  $M =$ number of discrete signal or voltage levels
Signal-to-noise ratio

The ratio $S/N$ is the “signal to noise” ratio.

- Units are often reported in “bels”, which is the logarithm (base 10) of the ratio
- Decibels: 10 times the above value
  - Example 35 dB = 3.5 bels

The ratio can also be expressed as a pure number:

- $S/N$ is 3162, meaning that the signal power is 3162 times the noise power.
Signal-to-Noise Ratio

- Ratio of the power in a signal to the power contained in the noise that’s present at a particular point in the transmission
- Typically measured at a receiver
- Signal-to-noise ratio (SNR, or S/N)

\[(SNR)_{dB} = 10 \log_{10} \frac{\text{signal power}}{\text{noise power}}\]

- A high SNR means a high-quality signal, low number of required intermediate repeaters
- SNR sets upper bound on achievable data rate
Shannon Capacity Formula

- When there is noise in the medium, capacity is given by
  \[ C = B \log_2 \left( 1 + \frac{S}{N} \right) \]

  Where: \( S/N \) = signal power/noise power

  - \( \text{SNR}_{dB} = 10 \log_{10} \text{SNR} \)

- Represents theoretical maximum that can be achieved.

- In practice, only much lower rates achieved.
  - Formula assumes white noise (thermal noise)
  - Impulse noise is not accounted for
  - Attenuation distortion or delay distortion not accounted for
Few things to remember

Using the calculator how to find

- \( \log_2 (x) = \frac{\ln (x)}{\ln (2)} \)
- \( \log_{10} (x) = \frac{\ln (x)}{\ln (10)} \)

How to convert S/N to/from dB

- \( y \, \text{dB} = 10 \log_{10} (S/N) \)
- \( 10^{\frac{y}{10}} \Rightarrow S/N \)

Also,

\[ y = \log_n x \quad \text{really means} \quad x = n^y \]
Example of Nyquist and Shannon Formulations

- Spectrum of a channel between 3 MHz and 4 MHz; \( \text{SNR}_{\text{dB}} = 24 \text{ dB} \)
  
  \[
  B = 4 \text{ MHz} - 3 \text{ MHz} = 1 \text{ MHz} 
  \]
  
  \[
  \text{SNR}_{\text{dB}} = 24 \text{ dB} = 10 \log_{10}(\text{SNR}) 
  \]
  
  \[
  \text{SNR} = 251 
  \]

- Using Shannon’s formula
  
  \[
  C = 10^6 \times \log_2(1 + 251) \approx 10^6 \times 8 = 8 \text{ Mbps} 
  \]
Example of Nyquist and Shannon Formulations

- How many signaling levels are required?

\[ C = 2B \log_2 M \]
\[ 8 \times 10^6 = 2 \times (10^6) \times \log_2 M \]
\[ 4 = \log_2 M \]
\[ M = 16 \]
Few things to remember

Using the calculator how to find

- $\log_2 (x) = \frac{\ln (x)}{\ln (2)}$
- $\log_{10} (x) = \frac{\ln (x)}{\ln (10)}$

How to convert S/N to/from dB

- $y \, \text{db} = 10 \log_{10} (S/N)$
- $10^{y/10} \Rightarrow S/N$

Also,

$y = \log_n x$ really means $x = n^y$
Nyquist Formula: Example 2

1. Given \( C \) in bits/second and \( L \) (a number). Find \( B \) in Hz?

Example: \( C = 1 \times 10^6 \text{ b/s} \) and \( L = 8 \)

\[
1 \times 10^6 = 2 \, B \log_2 (8)
\]

\[
B = 1.6 \times 10^7 \text{ Hz}
\]
**Nyquist Formula: Example 3**

\[ C = 2B \log_2 L \]

2. *Given C in bits/second and B in Hz. Find L?*

Example: \( C = 7 \times 10^6 \) b/s and \( B = 1.75 \times 10^6 \) Hz

\[
7 \times 10^6 = 2 \left(1.75 \times 10^6\right) \log_2 (L)
\]

\( L = 4 \)
Nyquist's Formula: Example 4

- \( C = 2B \log_2 L \)

3. Given \( B \) in Hz and \( L \) (in number). Find \( C \) in b/s?

Example: \( B = 6 \times 10^6 \) Hz and \( L = 12 \)

\[
C = 2 \times (6 \times 10^6) \log_2 (12)
\]

\[
C = 4.3 \times 10^7 \text{ b/s}
\]
Shannon’s Formula: Example 2

\[ C = B \log_2 \left( 1 + \frac{S}{N} \right) \]

1. Given \( C \) in b/s and \( B \) in Hz. Find \( S/N \)?

Example: \( C = 5 \times 10^6 \) b/s and \( B = 1.25 \times 10^6 \)

\[
5 \times 10^6 = (1.25 \times 10^6) \log_2 (1 + S/N)
\]

\( S/N = 16 \)

Another variation is to find \( S/N \) in decibels.

\[
S/N = 16 = \log_{10} (16) = 1.20 \text{ bels}
\]

\( S/N = 1.20 \times 10 = 12 \text{ decibels} \)
Shannon’s Formula: Example 3

\[ C = B \log_2 \left( 1 + \frac{S}{N} \right) \]

2. Given \( C \) in b/s and \( S/N \). Find \( B \) in Hz?

Example: \( C = 1 \times 10^9 \) Hz and \( S/N = 40 \) dB

First convert 40 dB to \( S/N \) ratio then apply Shannon’s Formula

\[ 1 \times 10^9 = B \log_2 \left( 1 + 1.0 \times 10^4 \right) \]

\[ B = 75.25 \times 10^6 \text{ Hz} \]
Shannon’s Formula: Example 4

\[ C = B \log_2 \left( 1 + \frac{S}{N} \right) \]

3. Given \( B \) in Hz and \( S/N \). Find \( C \) in b/s?

Example: \( B = 4 \times 10^9 \) Hz and \( S/N = 1023 \) (not in dB)

\[
\begin{align*}
C &= 4 \times 10^9 \log_2 (1 + 1023) \\
C &= 4 \times 10^{10} \text{ b/s}
\end{align*}
\]
Shannon’s Formula: Example 5

\[ C = B \log_2 \left( 1 + \frac{S}{N} \right) \]

4. Given \( B \) in Hz and \( S/N \) in dB. Find \( C \) in b/s?

Example: \( B = 8 \times 10^6 \) Hz and \( S/N = 45 \) dB

First convert decibels to \( S/N \) ratio \( \Rightarrow 10^{40/10} \)

\[ C = 8 \times 10^6 \log_2 (1 + 1.0 \times 10^4) \]
\[ C = 8 \times 10^{10} \times 1.33 \times 10^1 \]
\[ C = 1.64 \times 10^{12} \text{ b/s} \]
Question 1

What is the maximum theoretical capacity in bits per second of a coaxial cable band with a frequency spectrum of 50 MHz to 100 MHz and a signal to noise ratio of 40 dB?
Question 1

What is the maximum theoretical capacity in bits per second of a coaxial cable band with a frequency spectrum of 50 MHz to 100 MHz and a signal to noise ratio of 40 dB?

- Shannon’s formula:
- First, determine S/N:
  \[ 40 = 10 \times \log_{10}(S/N) \]
  \[ 4 = \log_{10}(S/N) \]
  \[ 1.0 \times 10^4 = S/N \]
- Determine B: \[ 1.0 \times 10^8 - 5.0 \times 10^7 = 5.0 \times 10^7 \text{ b/s} \]
Question 1

What is the maximum theoretical capacity in bits per second of a coaxial cable band with a frequency spectrum of 50 MHz to 100 MHz and a signal to noise ratio of 40 dB?

Now, apply the formula:

\[ C = B \log_2 \left( 1 + \frac{S}{N} \right) \]

\[ C = 5.0 \times 10^7 \times \log_2(1+1.0 \times 10^4) \]

\[ C = 5.0 \times 10^7 \times 1.33 \times 10^1 \]

\[ C = 6.64 \times 10^8 \text{ b/s (or 664 Mb/s)} \]
Question 2

We have a channel with a 1 MHz bandwidth. The ratio S/N is 63. What is the appropriate bit rate and number of signal levels?
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- We have a channel with a 1 MHz bandwidth. The ratio $S/N$ is 63. What is the appropriate bit rate and number of signal levels?
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\[
C = B \log_2 \left( 1 + \frac{S}{N} \right)
\]

- \[C = 1 \times 10^6 \log_2 (1+63)\]
  - \[= 1 \times 10^6 \log_2 (64)\]
  - \[= 1 \times 10^6 \times 6\]
  - \[= 6 \times 10^6 \text{ b/s}\]
Question 2

- We have a channel with a 1 MHz bandwidth. The ratio $S/N$ is 63. What is the appropriate bit rate and number of signal levels?
- Next, use the Nyquist formula to obtain the number of signal levels $L$:

$$C = 2B \log_2 L$$
Question 2

- We have a channel with a 1 MHz bandwidth. The ratio S/N is 63. What is the appropriate bit rate and number of signal levels?
- Next, use the Nyquist formula to obtain the number of signal levels L:

\[ C = 2B \log_2 L \]

\[ 6 \times 10^6 = 2 \times 1 \times 10^6 \log_2(L) \]

\[ 3 = \log_2(L) \]

\[ 8 = L \]

Therefore, 8 signal levels can be used to send bits at \( 6 \times 10^6 \) b/s over this channel.
Question 3

Suppose that an FM radio station is allocated 200 KHz of bandwidth, and wants to digitally broadcast stereo CD music which means that 16 bits need to be sent at a sample rate of 44,100 samples per second for each of 2 sound channels. How many signal levels are needed?

\[ C = 2B \log_2 L \]

\[ 44100 \times 16 \times 2 = 2B \log_2 L \]
Bandwidth Allocation

- Necessary to avoid interference between different radio devices
  - Microwave woven should not interfere with TV transmission
  - Generally a radio transmitter is limited to a certain bandwidth
    - 802.11 channel has 30MHz bandwidth
  - Power and placement of transmitter are regulated by authority
    - Consumer devices are generally limited to less than 1W power
Classifications of Transmission Media

- Transmission Medium
  - Physical path between transmitter and receiver

- Guided Media
  - Waves are guided along a solid medium
  - E.g., copper twisted pair, copper coaxial cable, optical fiber

- Unguided Media
  - Provides means of transmission but does not guide electromagnetic signals
  - Usually referred to as wireless transmission
  - E.g., atmosphere, outer space
Unguided Media

- Transmission and reception are achieved by means of an antenna
- Configurations for wireless transmission
  - Directional
  - Omnidirectional
General Frequency Ranges

- **Microwave frequency range**
  - 1 GHz to 40 GHz
  - Directional beams possible
  - Suitable for point-to-point transmission
  - Used for satellite communications

- **Radio frequency range**
  - 30 MHz to 1 GHz
  - Suitable for omnidirectional applications

- **Infrared frequency range**
  - Roughly, $3 \times 10^{11}$ to $2 \times 10^{14}$ Hz
  - Useful in local point-to-point multipoint applications within confined areas
Terrestrial Microwave

- Description of common microwave antenna
  - Parabolic "dish", 3 m in diameter
  - Fixed rigidly and focuses a narrow beam
  - Achieves line-of-sight transmission to receiving antenna
  - Located at substantial heights above ground level

- Applications
  - Long haul telecommunications service
  - Short point-to-point links between buildings
Satellite Microwave

Description of communication satellite

- Microwave relay station
- Used to link two or more ground-based microwave transmitter/receivers
- Receives transmissions on one frequency band (uplink), amplifies or repeats the signal, and transmits it on another frequency (downlink)

Applications

- Television distribution
- Long-distance telephone transmission
- Private business networks
Broadcast Radio

- Description of broadcast radio antennas
  - Omnidirectional
  - Antennas not required to be dish-shaped
  - Antennas need not be rigidly mounted to a precise alignment

- Applications
  - Broadcast radio
    - VHF and part of the UHF band; 30 MHz to 1GHz
    - Covers FM radio and UHF and VHF television
Multiplexing

- Capacity of transmission medium usually exceeds capacity required for transmission of a single signal
- Multiplexing - carrying multiple signals on a single medium
  - More efficient use of transmission medium
Multiplexing
Reasons for Widespread Use of Multiplexing

- Cost per kbps of transmission facility declines with an increase in the data rate
- Cost of transmission and receiving equipment declines with increased data rate
- Most individual data communicating devices require relatively modest data rate support
Multiplexing Techniques

- Frequency-division multiplexing (FDM)
  - Takes advantage of the fact that the useful bandwidth of the medium exceeds the required bandwidth of a given signal

- Time-division multiplexing (TDM)
  - Takes advantage of the fact that the achievable bit rate of the medium exceeds the required data rate of a digital signal
Frequency-division Multiplexing
Time-division Multiplexing