

# Neoclassical Growth Economy with Labor-Leisure Choice

# Household Problem

- ▶ Preferences

$$\max E \left\{ \sum_{t=0}^{\infty} \beta^t (1+\eta)^t [(\ln(c_t) + \phi \ln(1-l_t)) \right\},$$

- ▶ Constraints

$$c_t + i_t = w_t l_t + r_t k_t,$$
$$(1+\gamma)(1+\eta)k_{t+1} = (1-\delta)k_t + i_t.$$

## Problem of the Firm

- ▶ Neoclassical production function

$$y_t = e^{z_t} k_t^\alpha l_t^{1-\alpha}$$

- ▶ By profit maximization

$$r_t = \alpha e^{z_t} k_t^{\alpha-1} l_t^{1-\alpha},$$
$$w_t = 1 - \alpha e^{z_t} k_t^\alpha l_t^{-\alpha}.$$

## Equilibrium Conditions

$$\frac{1+\gamma}{c_t} = \frac{\beta (1 + \alpha e^z k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} - \delta)}{c_{t+1}}; \quad (1)$$

$$\frac{\phi c_t}{(1-\phi)(1-l_t)} = (1-\alpha) \frac{y_t}{l_t}; \quad (2)$$

$$c_t + i_t = y_t; \quad (3)$$

$$y_t = e^z k_t^\alpha l_t^{1-\alpha}; \quad (4)$$

$$i_t = (1+\gamma)(1+\eta)k_{t+1} - (1-\delta)k_t; \quad (5)$$

## Deterministic Steady State

- ▶ The equilibrium conditions imply a steady state:

$$1 + \gamma = \beta \left( 1 + \alpha e^z k^{\alpha-1} l^{1-\alpha} - \delta \right); \quad (6)$$

$$\frac{\phi}{(1-\phi)} = (1-\alpha) e^z k^\alpha l^{-\alpha}; \quad (7)$$

$$c + (1+\gamma)(1+\eta)k - (1-\delta)k = e^z k^\alpha l^{1-\alpha}; \quad (8)$$

- ▶ Or simplifying

$$\frac{1+\gamma}{\beta} + \delta - 1 = \alpha \frac{y}{k}; \quad (9)$$

$$(1-\alpha) \frac{y}{c} = \frac{\phi}{1-\phi} \frac{l}{1-l}; \quad (10)$$

$$\frac{i}{k} = \delta - 1 + (1+\gamma)(1+\eta); \quad (11)$$