Izmir University of Economics
Department of Economics
Econ 533: Quantitative Methods and Econometrics
Fall 2014
Homework 2

This homework is due on Thursday December 18 at the beginning of class. Do not hesitate to contact me via e-mail if you have specific questions about the homework. Do the questions ON YOUR OWN.

1. Let f be a function of two variables given by

$$f(x, y) = (x^2 - y)^2 + x^3 - 3x^2 \qquad \text{for all } x \text{ and } y$$

   a. Calculate the first and second order partial derivatives of f.

   b. Find the maximum and minimum points of f and classify them by means of the second-derivative test.

   c. Does f have any global extreme values?

2. The demands for a monopolist's two products are determined by the equations

$$p = 20x^{-1/2}, \qquad q = 51 - 0.5y$$

   where p and q are prices per unit of the two goods, and x and y are the corresponding quantities. The cost of producing and selling x units of the first good and y units of the other is $c(x, y) = x + y$.

   a. Find the monopolist's profit $\pi(x, y)$ from producing and selling x units of the first good and y units of the other.

   b. Find the values of x and y that maximize $\pi(x, y)$. Verify that you have found the maximum profit.

   c. What is the marginal revenue in each of the two markets? What are the marginal revenues when profit is maximized?

3. Consider the problem

$$\max \text{ (min)} \quad f(x,y) = x^2 + y^2$$

$$\text{subject to} \quad g(x,y) = 5x^2 + 6xy + 5y^2 = 1$$

   a. Use Langrange's method to solve the problem.

   b. The constraint curve is an ellipse in the xy-plane. Give a geometric interpretation to the problem.

4. A monopolist supplies two markets, one at home, the other abroad. The demand functions are

$$\begin{aligned} q_1 &= 10 - p_1 \\ q_2 &= 5 - 0.5p_2 \end{aligned}$$

   where q_1 denotes home sales and q_2 foreign sales. The firm's total cost function is

$$C = 0.5(q_1 + q_2)^2$$

   a. Find the profit maximizing output and prices (No arbitrage between the markets is possible).

   b. Suppose now that price regulation is imposed in the home market, in the form of a maximum price of \$b. What is the effect of this on prices, outputs and profit? Explain your results.

   Use solver to solve the following questions:

5. A person has utility function $u(x,y) = 100xy + x + 2y$ Suppose that the price per unit of x is 2 dollars, and that the price per unit of y is 4 dollars. The person receives 1000 dollars that all has to be spent on the two commodities x and y. Solve the utility maximization problem.

6. min $f(x,y) = x^2 + y^2$ subject to $g(x,y) = x + 2y = 4$

7. max $f(x,y) = x^2 + 3xy + y^2$ subject to $g(x,y) = x + y = 100$