

Izmir University of Economics  
Department of Economics  
Econ 533: Quantitative Methods and Econometrics  
Fall 2015  
**Homework 1**

This homework is due on Thursday November 12 at the beginning of class. Do not hesitate to contact me via e-mail if you have specific questions about the homework.

1. The price per unit obtained by a firm that sells  $x \geq 0$  units is  $p = 144 - x$ , while the cost is  $C(x) = \frac{1}{3}x^3 - 6x^2 + 160x$ .
  - a. Show that the marginal cost  $C'(x)$  is always positive.
  - b. Write down the profit function.
  - c. Find the value of  $x$  that maximizes profits.
2.
  - a. A firm is taking prices as given The price per unit sold is 1000, and the cost function is  $C(x) = 0.01x^3 - 3x^2 + 1108x + 960$ , where  $x$  is the number of units produced and sold. Find the profit function,  $\pi(x)$ ,  $x \geq 0$ .
  - b. The profit function has two stationary (critical) points. Which of them maximizes profit? Sketch the graph of the profit function. Where is the point of inflection? Give an economic interpretation of the inflection point.
3. An ice-cream lover has a total of 10 dollars to spend one evening. The price of ice-cream is  $p$  dollar per pint. The person's preferences for buying  $q$  pints of ice-cream, and then having  $(10 - pq)$  dollars to spend on other items, are represented by the utility function

$$U(q) = \sqrt{q} + 2\sqrt{10 - pq}$$

- a. Find the first-order condition for a utility maximizing quantity of ice-cream,  $q^*$ .

- b. Solve the first-order condition derived in (a) in order to express  $q^*$  as a function of  $p$ .
  - c. What guarantees that your answer to (a) is really a maximum?
4. Estimate any extrema of the functions given by

a.

$$f(x) = 4x^3 - 28x^2 + 40x + 2\sqrt{x}$$

b.

$$f(x) = x^2 - x^5$$

c.

$$f(x) = -8x^3 + 9x^2 + 6x - 2$$

5. Suppose a firm has the following demand equation

$$Q = 1,000 - 3,000P + 10A$$

where  $Q$  is quantity demanded,  $P$  is product price in dollars and  $A$  is the advertising expenditure. Assume for the following questions that  $P = \$3$  and  $A = \$2,000$ .

- a. Suppose the firm dropped the price to \$2.50. Would this be beneficial? Explain. Illustrate your answer with the use of a demand schedule and a demand curve.
  - b. Suppose the firm raised the price to \$4 while increasing its advertising expenditure \$100. Would this be beneficial? Explain. Illustrate your answer with the use of a demand schedule and a demand curve.
6. a. Find absolute maximum and minimum values of the function, if they exist, over  $[0, \infty)$ :

$$f(x) = \frac{x^2}{4 + x^2}$$

- b. Find absolute maximum and minimum values of the function, if they exist, over  $(-\infty, 0]$ :

$$f(x) = \frac{4x^2}{1 - x^3}$$

- c. Total cost, in dollars, of producing  $x$  units of a product is given by:

$$C(x) = 90x + 90\sqrt{x} + \frac{\sqrt{x^3}}{90}.$$

How many units should be produced to minimize average cost?

7. By producing and selling  $Q$  units of commodity a firm earns total revenue  $R(Q) = -0.0016Q^2 + 44Q$  incurs cost  $C(Q) = 0.0004Q^2 + 8Q + 64000$ . What production level  $Q^*$  maximizes profits?