# Izmir University of Economics
# Department of Economics
# Econ 533: Quantitative Methods and Econometrics
# Fall 2014
# **Homework 1**

This homework is due on Thursday November 20 at the beginning of class. Do not hesitate to contact me via e-mail if you have specific questions about the homework.

1. Consider the following macroeconomic model

$$\begin{aligned} Y &= C + I \\ C &= f(Y - T) \\ T &= \alpha + \beta Y \end{aligned}$$

where $Y$ is national income, $C$ is consumption, $T$ denotes taxes, $\alpha$ and $\beta$ are positive constants. Assume that $f' \in (0, 1)$ and $\beta \in [0, 1]$.

   **a.** Derive equation $Y = f((1 - \beta)Y - \alpha) + I$

   **b.** Differentiate the equation obtained in (a) implicitly w.r.t $I$ and find an expression for $dY/dI$.

   **c.** Examine the sign of $dY/dI$.

2. The price per unit obtained by a firm that sells $x \geq 0$ units is $p = 144 - x$, while the cost is $C(x) = \frac{1}{3}x^3 - 6x^2 + 160x$.

   **a.** Show that the marginal cost $C'(x)$ is always positive.

   **b.** Write down the profit function.

   **c.** Find the value of $x$ that maximizes profits.

3. **a.** A firm is taking prices as given The price per unit sold is 1000, and the cost function is $C(x) = 0.01x^3 - 3x^2 + 1108x + 960$, where $x$ is the number of units produced and sold. Find the profit function, $\pi(x)$, $x \geq 0$.

**b.** The profit function has two stationary points. Which of them maximizes profit? Sketch the graph of the profit function. Where is the point of inflection? Give an economic interpretation of the inflection point.

4. An ice-cream lover has a total of 10 dollars to spend one evening. The price of ice-cream is $p$ dollar per pint. The person's preferences for buying $q$ pints of ice-cream, and then having $(10 - pq)$ dollars to spend on other items, are represented by the utility function

$$U(q) = \sqrt{q} + 2\sqrt{10 - pq}$$

   **a.** Find the first-order condition for a utility maximizing quantity of ice-cream, $q^*$.

   **b.** Solve the first-order condition derived in (a) in order to express $q^*$ as a function of $p$.

   **c.** What guarantees that your answer to (a) is really a maximum?

   **d.** Express the elasticity of demand for ice-cream as a function of the price $p$ dollar per pint. When the price is 2.50 dollars per pint, what is the price elasticity of the person's demand for ice-cream?

5. A mining company extracts metal ore worth $\$A$ of revenue in year 0. Its costs per year are $\$K$, a constant. But the company expects its revenue to decline by $p$ percent each year. Its mining activity will continue as long as the yearly revenue exceeds the yearly costs $K$.

   **a.** What is the revenue in year $n$? For how many years $n^*$ does the company operate?

   **b.** Find the total profit in the period from year 0 to $n^*$.

   **b.** Put $A$=7 000 000, $K$=5 000 000, and $p = 2$, answer the questions in this case.

6. **a.** How much must you deposit today to have 12 000 at the end of each year for 10 years, when the interest rate is 6 percent per year?

   **b.** How much must you deposit today to have 1000 at the end of each month for 10 years, when the interest rate is 6 percent per year?

   **c.** 48 months from now you plan a trip costing 60 000. To cover the costs you plan to save a fixed amount $a$ each month in a bank account, offering 6 percent yearly interest. How much must you save each month?

7. When investing in a savings account, which of the following offers is better: 5 percent per annum with interest paid monthly; or 4.9 percent per annum with interest paid daily?

   Use the Solver to answer the following questions:

8. Consider the case where a kind aunt leaves a nephew a yearly stipend of £1000 as long as he is at university, but limited to a maximum of 10 years, and £10 000 the year he graduates. The annual interest rate is 5 percent. The aunt hopes that this scheme will give the bright, but lazy student an incentive to work hard at university and graduate as soon as possible so that he can receive his graduation bonus of £10 000. The student is bright enough to complete his education whenever he wants to.

   **a.** Calculate the total present value of the aunt's stipend for all possible graduation dates. Does the stipend provide the student with an incentive to complete his education as soon as possible?

   **b.** If the interest rate increased to 15 percent, would this provide more incentive to graduate as soon as possible.

   **c.** Use the Solver to find the interest rate that makes the present value of graduating the first year equal to the present value of graduating after 10 years.

9. By producing and selling Q units of commodity a firm earns total revenue $R(Q) = -0.0016Q^2 + 44Q$ incurs cost $C(Q) = 0.0004Q^2 + 8Q + 64000$. What production level $Q^*$ maximizes profits?