Izmir University of Economics
Department of Economics
Econ 533: Quantitative Methods and Econometrics
Fall 2013
Homework 1

This homework is due on Thursday November 13 at the beginning of class. Do not hesitate to contact me via e-mail if you have specific questions about the homework.

1. Let f be a function of two variables given by

$$f(x, y) = (x^2 - y)^2 + x^3 - 3x^2 \qquad \text{for all } x \text{ and } y$$

   a. Calculate the first and second order partial derivatives of f.

   b. Find the maximum and minimum points of f and classify them by means of the second-derivative test.

   c. Does f have any global extreme values?

2. The demands for a monopolist's two products are determined by the equations

$$p = 20x^{-1/2}, \qquad q = 51 - 0.5y$$

   where p and q are prices per unit of the two goods, and x and y are the corresponding quantities. The cost of producing and selling x units of the first good and y units of the other is $c(x, y) = x + y$.

   a. Find the monopolist's profit $\pi(x, y)$ from producing and selling x units of the first good and y units of the other.

   b. Find the values of x and y that maximize $\pi(x, y)$. Verify that you have found the maximum profit.

   c. What is the marginal revenue in each of the two markets? What are the marginal revenues when profit is maximized?

3. Consider the problem

$$\max\ (\min) \quad f(x,y) = x^2 + y^2$$

$$\text{subject to} \quad g(x,y) = 5x^2 + 6xy + 5y^2 = 1$$

   a. Use Langrange's method to solve the problem.

   b. The constraint curve is an ellipse in the xy-plane. Give a geometric interpretation to the problem.

   c. Determine the approximate changes in the maximum amd minimum values of $f(x,y)$ if the constraint $g(x,y) = 1$ is replaced by $g(x,y) = 1.1$.

4. Consider the problem

$$\max\ (\min) \quad f(x,y,z) = x^2 + y^2 + z$$

$$\text{subject to} \quad g(x,y,z) = x^2 + 2y^2 + 4z^2 = 1$$

   (The graph of the constraint is the surface of an ellipsoid in R^3. This set is closed and bounded.)

   a. Solve the problem.

   b. Estimate the maximum value of f on the constraint set $x^2+2y^2+4z^2 = 1.02$.

5. A monopolist supplies two markets, one at home, the other abroad. The demand functions are

$$\begin{aligned} q_1 &= 10 - p_1 \\ q_2 &= 5 - 0.5p_2 \end{aligned}$$

   where q_1 denotes home sales and q_2 foreign sales. The firm's total cost function is

$$C = 0.5(q_1 + q_2)^2$$

   a. Find the profit maximizing output and prices (No arbitrage between the markets is possible).

b. Suppose now that price regulation is imposed in the home market, in the form of a maximum price of $b. What is the effect of this on prices, outputs and profit? Explain your results.

6. **a.** Maximize $f(x, y, z) = yz + xz$ subject to $y^2 + z^2 = 1$ and $xz = 3$

   b. Check the second order conditons for the solutions of the first order conditions.

7. A consumer has the utility function

$$u = x_1 x_2$$

and she faces the money-income constraint

$$2x_1 + 3x_2 \leq 100$$

and the time constraint

$$x_1 + 4x_2 \leq 80$$

Solve for her utility-maximizing consumption bundle.

8. Consider the following minimization problem

$$\min \quad x^2 - 2y$$

$$\text{subject to} \quad x^2 + y^2 \leq 1, x \geq 0, y \geq 0$$

   a. Write down the Langrangian and the necessary Kuhn-Tucker conditions for the problem

   b. Find the solution to the problem.

Hint: Please notice that this is a minimization problem. First write out the Kuhn-Tucker formulation for a constrained minimization problem.

9. Consider the problem

$$\max \quad x + ay$$

$$\text{subject to} \quad x^2 + y^2 \leq 1, \quad x + y \geq 0$$

   a. Find the solution for all vales of constant a.

   b. What is the approximate change in the maximum value of the objective function, if the constraint set is changed to $x^2 + 0.9y^2 \leq 1$?