Izmir University of Economics
Econ 533: Quantitative Methods and Econometrics

Constrained Optimization

The Lagrange Multiplier Method

- To find only possible solutions of the problem

$$max(min) f(x,y) \quad subject\ to \quad g(x,y) = c \tag{1}$$

Proceed as follows

(I) Write down the Lagrangian

$$L(x,y) = f(x,y) - \lambda(g(x,y) - c) \tag{2}$$

where λ is a constant

(II) Differentiate L with respect to x and y, and equate the partial derivatives to 0.

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(III) The two equations in (II), together with the constraint yield the following three equations:

$$\begin{aligned} L_1'(x,y) &= f_1'(x,y)\lambda(g_1'(x,y)-c)=0 \\ L_2'(x,y) &= f_2'(x,y)\lambda(g_2'(x,y)-c)=0 \\ g(x,y) &= c \end{aligned}$$

(IV) Solve these equations simultaneously for the three unknowns x, y, and λ.

- The conditions in (III) are called the first-order conditions.

Examples

- Example 1: A consumer has the utility function $u(x, y) = xy$ and faces the budget constraint $2x + y = 100$. Find the only solution candidate to the consumer demand problem

$$max\ xy \quad subject\ to \quad 2x + y = 100$$

- Example 2: A single product firm intends to produce 30 units of output as cheaply as possible. By using K units of capital and L units of labor, it can produce $\sqrt{K} + L$ units. Suppose the prices of capital and labour are, respectively, 1 and 20. The firm's problem is then

$$min\ K + 20L \quad subject\ to \quad \sqrt{K} + L = 30$$

Find the optimal choices of K and L.

Examples

- Example 3: Examine the general utility maximizing problem with two goods:

$$max \ \ u(x, y) \quad subject \ \ to \quad px + qy = m$$