Izmir University of Economics
Financial Economics
Econ 533: Quantitative Methods and Econometrics
Fall 2014

Mid-Term Exam 2

This take-home exam is due on December 25 at the beginning of class.

1. A consumer's demands x, y for two different goods are chosen to maximize the utility function

$$U(x, y) = \sqrt{x} + \sqrt{y} \qquad (x \geq 0, y \geq 0)$$

   subject to the budget constraint $px + qy = m$ (where $p, q, m > 0$).

   a. Write out the Lagrangian for the constrained maximization problem.

   b. Write out the first order conditions for a constrained maximum at (x^*, y^*).

   c. Find the utility maximizing demands for both goods and the Lagrangian multiplier as functions of the three variables (p, q, m).

   d. Find $\partial U^*/\partial m$ and comment on your answer.

2. Mathematica Records needs to produce 100 gold records at one or more of its three studios. The cost of producing x records at studio 1 is $10x$; the cost of producing y records at studio 2 is $2y^2$; the cost of producing z records at studio 3 is $z^2 + 8z$.

   a. Formulate the problem of producing the 100 records at minimum cost.

   b. What is the Lagrangian associated with your formulation in (a)?

   c. Solve this Lagrangian. What is the optimal production plan?

   d. What is the marginal cost of producing one extra gold record?

e. Union regulations require that exactly 60 hours of work be done at studios 2 and 3 combined. Each gold record requires 4 hours at studio 2 and 2 hours at studio 3. Formulate the problem of finding the optimal production plan, give the Lagrangian, and give the set of equations that must be solved to find the optimal production plan. It is not necessary to actually solve the equations.

3. A two-product firm faces the demand and cost functions below:

$$\begin{aligned} Q_1 &= 40 - 2P_1 - P_2 \\ Q_2 &= 35 - P_1 - P_2 \\ C &= Q_1^2 + 2Q_2^2 + 10 \end{aligned}$$

a. Find the output levels that satisfy the first order conditions for maximum profit.

b. Check the second order condition. Can you conclude that this problem possesses a unique absolute maximum?

c. What is the maximal profit?

4. A firm has the production function $Q = F(K, L) = K^{1/2}L^{1/4}$ for $K > 0$ and $L > 0$, and chooses its inputs K of capital and L of labor in order to maximize its profit $\pi = PQ - rK - wL$.

(a) Show that π is concave.

(b) Write out the first-order conditions for the profit maximization problem.

(c) Show that $rK = \frac{1}{2}PQ$ and $wL = \frac{1}{4}PQ$ at the maximum.

(d) Solve the first-order conditions for the firm's input demands as functions $K^*(P, r, w)$ and $L^*(P, r, w)$.

(e) Find the firm's optimal output and maximum profit as a functions $Q^(P, r, w)$ and $\pi^*(P, r, w)$ respectively.

(f) Use the answers to (d) and (e) in order to verify that the first-order partial derivatives of $\pi^*(P, r, w)$ satisfy $\partial\pi^*/\partial P = Q^*$, $\partial\pi^*/\partial r = -K^*$, and $\partial\pi^*/\partial r = -L^*$ Then give economic interpretations of these three equalities.